Page
Date

Inverse Kinematics

Given: $\underline{\underline{H_d}}$ = position & Orientation of EE

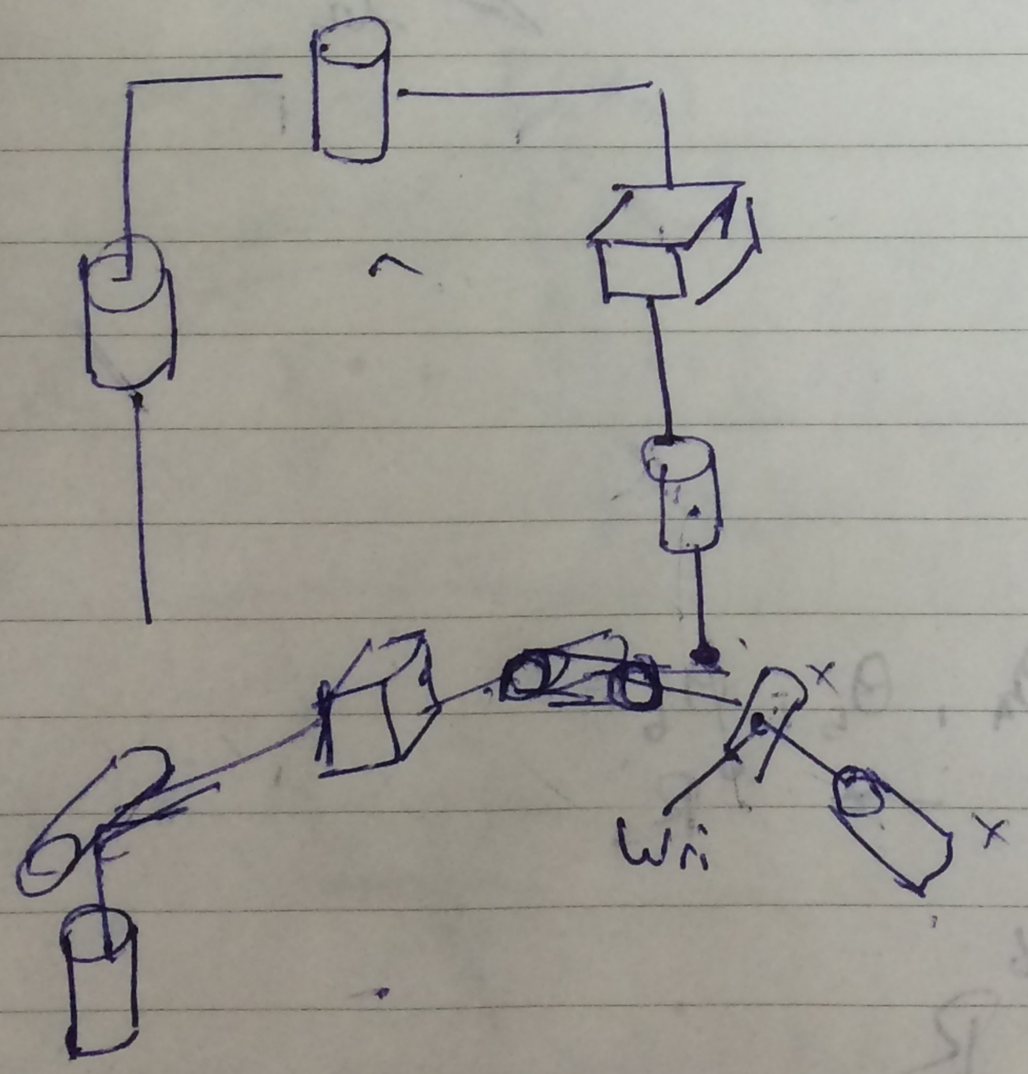
Required: (q_k), Joint Variables to achieve g (desired state)

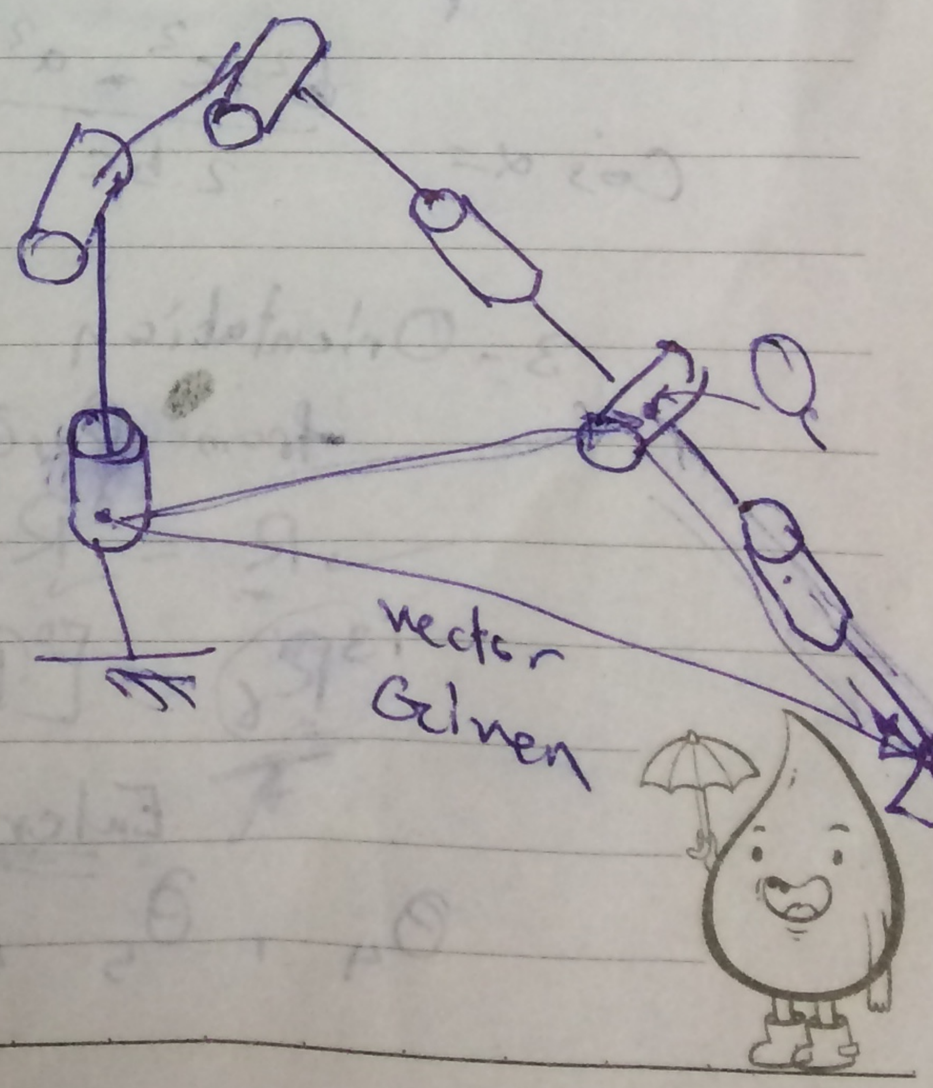
$$g = [x_{EE}, y_{EE}, z_{EE}, \theta_x, \theta_y, \theta_z]^T$$

→ Closed Form (Analytical - Geometric)
→ Numerical Solutions (Iterative)
→ Artificial Intelligence

Closed Form: Geometric approach

- Kinematics Decoupling
  - Spherical wrist in 6DoF manipulator
    ex: elbow manipulator
    cylinderical manipulator spherical wrist
    → SCARA





Date

steps for Inverse kinematics using Geometric approach

Given ${}^{base}H_{EE_desire}$: O_{EE} R

Steps

1- Find wrist center

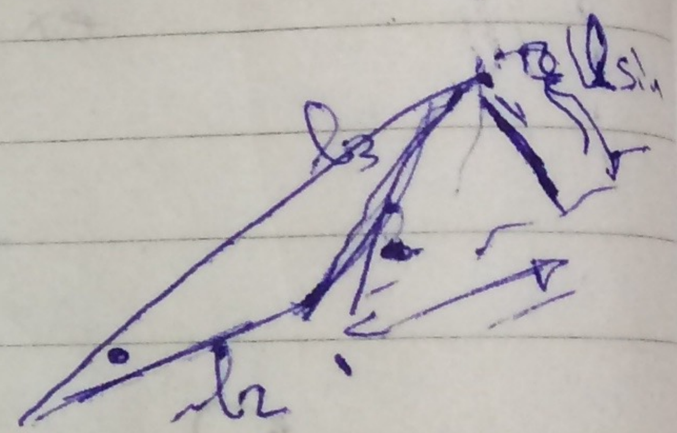
- wrist center position depends on first three joint variable

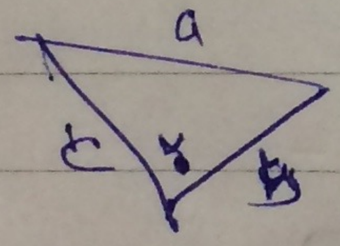
$$O_c = O_{EE} - d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2- From $O_c \rightarrow q_1, q_2, q_3$

Trigonometric equation

$$\begin{cases} \theta_1 = f_1(x_c, y_c, z_c) \\ \theta_2 = f_2(x_c, y_c, z_c) \\ \theta_3 = f_3(x_c, y_c, z_c) \end{cases}$$





$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

3- Orientation : $\theta_4, \theta_5, \theta_6$

from $\theta_1, \theta_2, \theta_3 \rightarrow {}^0R_3$

$R = {}^0R_3 \, {}^3R_6$

${}^3R_6 = [{}^0R_3]^{-1} R$

Euler angles

$\theta_4, \theta_5, \theta_6$

Glaxey

2 Numerical Methods

- Given g: Required q
- Forward kinematics f(q)

desire

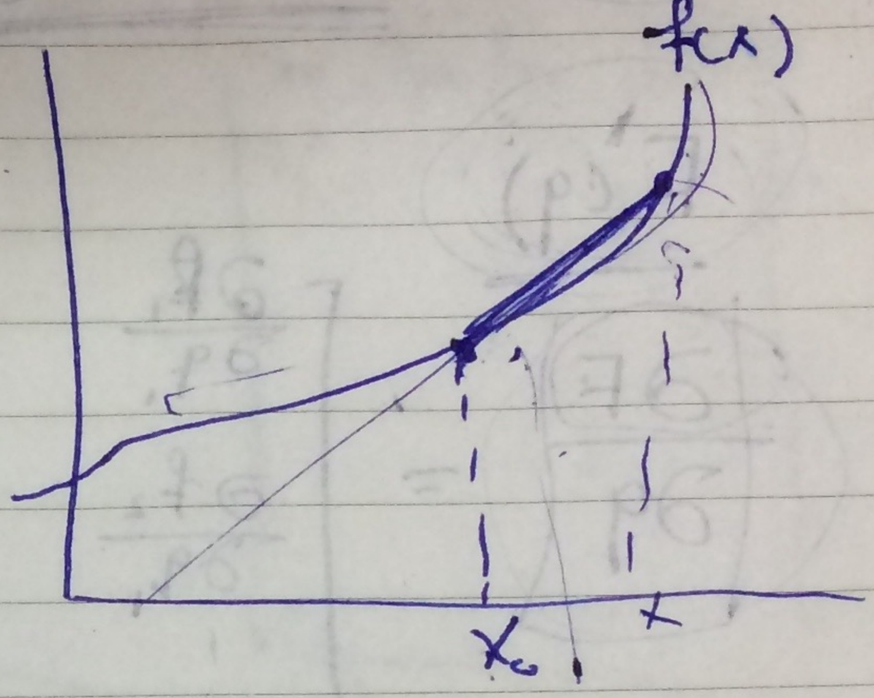
$$f(q) - g = 0 \quad \text{---(1)}$$

choose q to achieve (1)

Tylor

$$f(x) = f(x_0) + f'(x_0)(x - x_0)$$

↳ equals 0

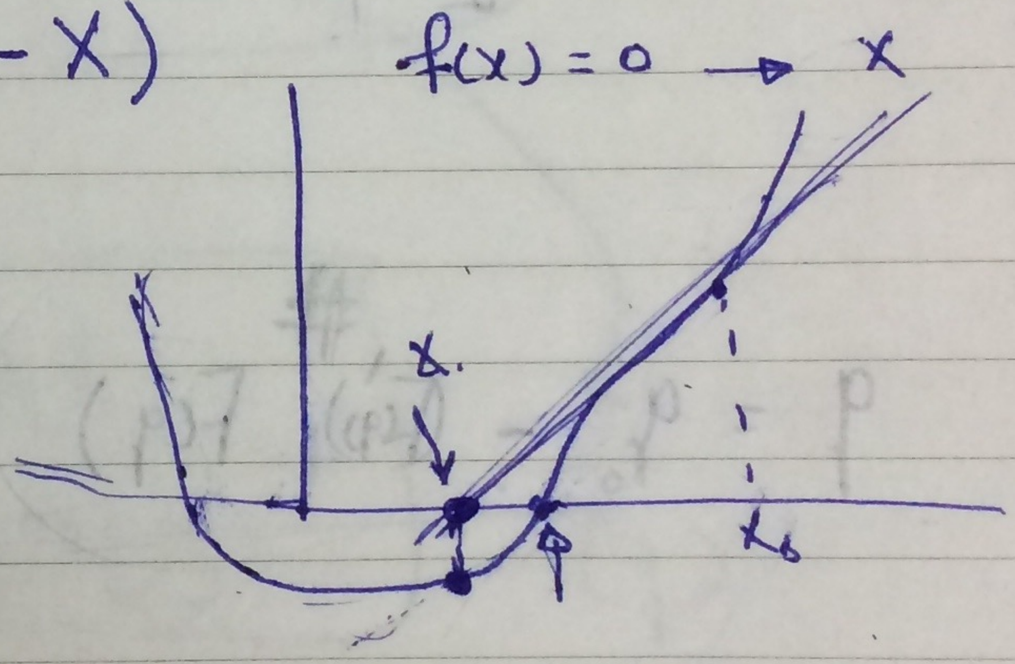


$$0 = f(x_0) + f'(x_0)(x - x_0)$$

$$= f(x_0) = f'(x_0)(x_0 - x)$$

$$x_0 - x = \frac{f(x_0)}{f'(x_0)}$$

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$ iterative



$$F(q) = f(q) - g \quad \text{(forward kinematics)}$$

$$q = q_0 - \frac{F(q)}{F'(q)}$$ vector

Glaxey

$\theta_x, \theta_y, \theta_z$

$$T_4 = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$f(q) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix}$$

3-

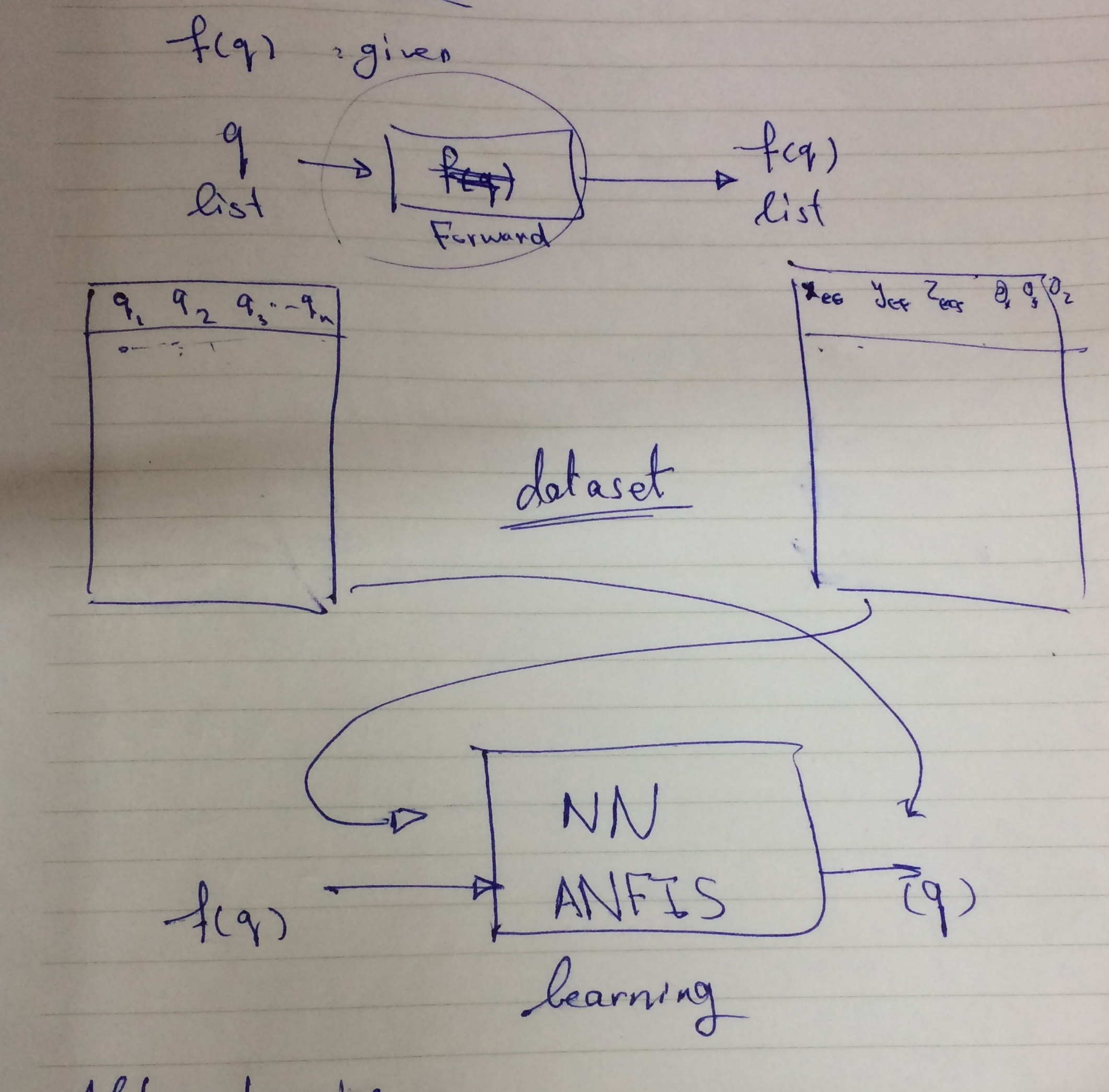
$$F(q) = f(q) - g$$

$$F'(q)$$

$$\frac{\partial F}{\partial q} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \cdots & \frac{\partial f_1}{\partial q_n} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \cdots & \frac{\partial f_2}{\partial q_n} \\ \vdots & & & \\ & & & \frac{\partial f_6}{\partial q_n} \end{bmatrix}$$

Jacobians

$$q = q_0 - (F'(q))^{\#} F(q)$$

Page
Date

3- Artificial Intelligence